BENDING STRESSES IN BEAMS

* Assumptions in Bending stresses
* Theory of simple bending
* Expression for bending stresses
* Neutral axis and moment of resistance
* Bending stresses in symmetrical sections.
* Section modulus
* Section modulus of various sections.

Assumptions in Bending stresses

(1) Beam material should be perfectly homogeneous.

(2) Under elastic limit, stresses should be proportional to strain.

(3) Modulus of Elasticity should be same for fibers under compression and tension.

(4) The transverse section of the beam should be plane before and after bending.

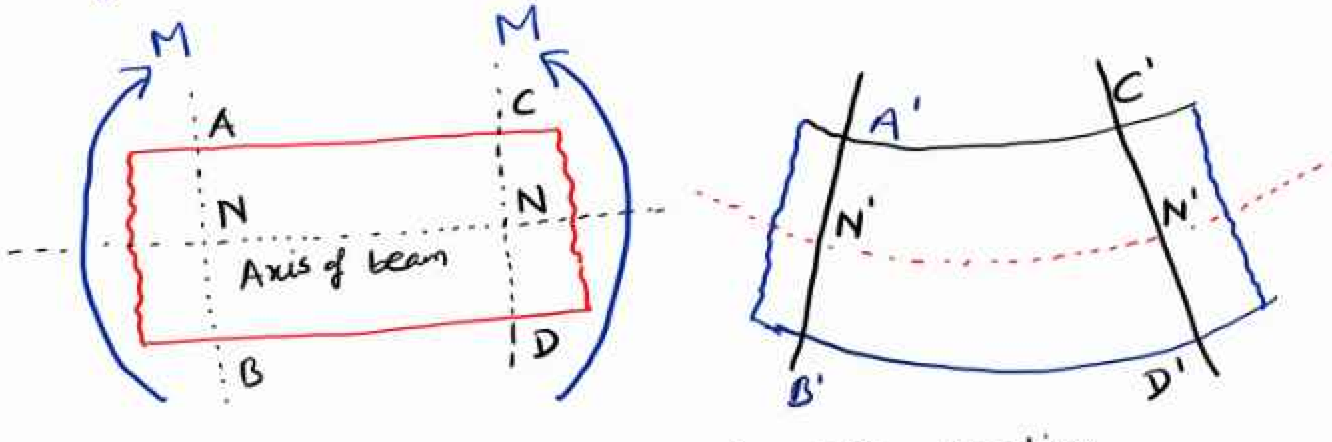
(5) There is no resultant pull and push on the cross-section of the beam after bending.

(6) The transverse section of the beam is symmetrical about a line passing through the centre of gravity in the plane of bending.

(7) The radius of curvature of the beam is large when compare with its transverse dimensions.

Theory of Simple Bending

Figure shows a part of the beam subjected to a simple bending. Consider a small length δx of this part of beam.



(a) Before Bending (b) After bending

Here, the section AB & CD are normal to the axis of beam N-N. Due to the action of bending moment, the small length δx will deform which is shown in figure (b). It means that the layer of beam is changes its length after bending

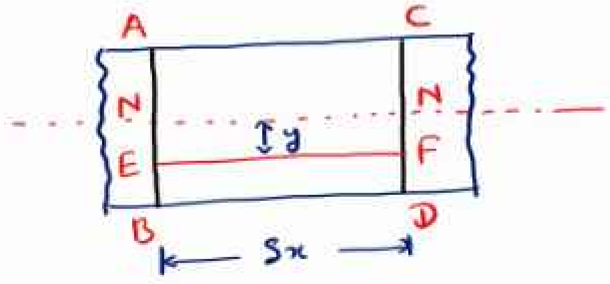
Top layer AC will deform and become A'C' and bottom layer will deform from BD to B'D'.

In result, some layer will shorten its length and some layer will be elongated after simple bending.

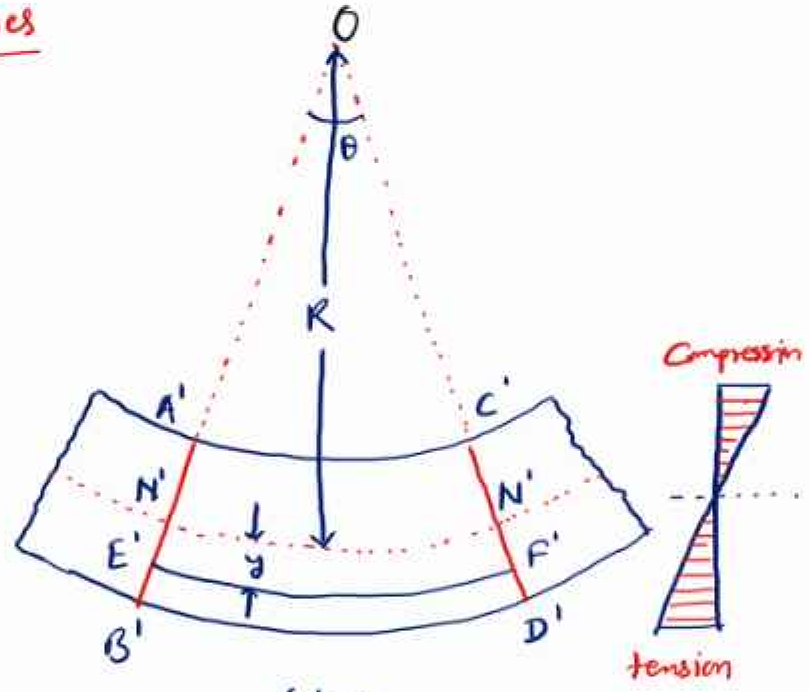
A layer NN and NN' between top and bottom layer will be neither shortened nor elongated is called as neutral layer (or) neutral surface (or) neutral axis.

There are two layers above and below neutral axis will be shortened and elongated. The shortened layer above neutral axis (or) layer will be under compression and a lower layer below neutral axis (or) layer will be under tension.

Expression for bending stresses



(a)



(b)

Figure shows a small length δx of a beam subjected to a simple bending. Due to the action of simple bending a small length δx will deform.

Let R is the radius of neutral layer $N'N'$

θ is angle made at O by $A'B'$ & $C'D'$

Consider a layer EF at a distance y below the neutral layer NN. After bending this layer elongated to $E'F'$.

Original length of layer $= EF = \delta x$

length of neutral layer $= NN = \delta x$

After bending length of neutral layer $N'N'$ will remain unchanged but the length of $E'F'$ will increase.

Hence, $N'N' = NN = \delta x$

From figure(b),

$$N'N' = R\theta$$

$$E'F' = (R+y)\theta$$

$$N'N' = NN = \delta x$$

$$\delta x = R\theta$$

Increase in the length of layer EF

$$= E'F' - EF = (R+y)\theta - R\theta$$
$$= R\theta + y\theta - R\theta$$
$$= y \times \theta$$

Now, define strain in layer EF

So, strain in layer EF $= \dfrac{\text{Increase in length}}{\text{Original length}}$

$$= \frac{y \times \theta}{EF} = \frac{y\theta}{R\theta} = \frac{y}{R} \quad \text{—— (1)}$$

As R is constant, hence the strain in layer is proportional to its distance from the neutral axis. The above expression shows the linear variation of strain along depth of beam.

Now, Stress variation

Let, σ = Stress in the layer EF
E = young's modulus of the beam

So, $E = \dfrac{\text{Stress in the layer EF}}{\text{Strain in the layer EF}}$

$$E = \frac{\sigma}{\left(\frac{y}{R}\right)}$$

$$\sigma = E \times \left(\frac{y}{R}\right) = \frac{E}{R} \cdot y$$

$$\boxed{\sigma = \frac{E}{R} \cdot y} \quad \text{—— (2)}$$

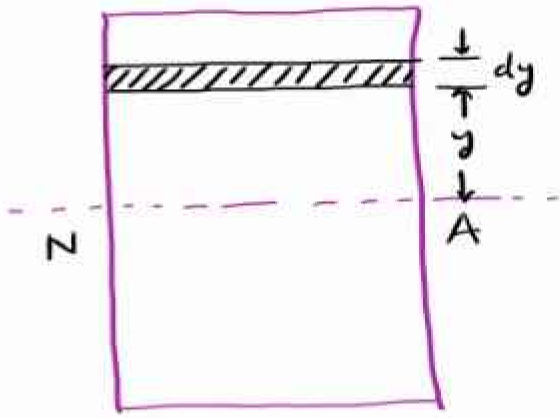
Since E & R are constant, therefore stress in any layer is directly proportional to the distance of layer from the neutral layer.

The above expression (2) can be written in the form

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

Neutral axis and moment of Resistance

The neutral axis is defined as the line of intersection of the neutral layer with the transverse section.



If the section of the beam is subjected to pure sagging moment then the stresses will be compressive at any point above the neutral axis and tensile below the neutral axis. There is no stresses at neutral axis.

From eqn ② the stresses at a distance y from the neutral axis.

$$\sigma = \frac{E}{R} \cdot y$$

From the above figure, NA is neutral axis of the section. Consider a small layer at a distance y from the neutral axis. Let dA is area of the layer.

Force on the layer = Stress on layer x area of layer

$$= \sigma \cdot dA$$

$$= \frac{E}{R} \cdot y \times dA \quad \text{——— } ③$$

Total force on the beam section is obtained by integrating the above equation

∴ Total force on the beam section =

$$\int \frac{E}{R} \cdot y \cdot dA$$

But for pure bending there is no force on the section of beam.

$$\therefore \frac{E}{R} \int y \cdot dA = 0 \Rightarrow \int y \times dA = 0$$

Here, $y \cdot dA$ represents the moment of area dA about neutral axis. Hence, $\int y \times dA$ represents the moment of entire area of the section about neutral axis. But it is known that moment of any area about an axis passing through its centroid, is also equal to zero. Hence neutral axis coinsides with the centroidal axis.

Moment of Resistance

Due to pure bending, the layer above the neutral axis are subjected to compressive stresses whereas layers below the neutral axis are subjected to tensile stresses. Due to these stresses, the forces acting on the layers. These forces will have moment about neutral axis. The total moment of these forces about the neutral axis for a section is known as moment of resistance of the section.

The force on the layer is given by

$$\text{eq}^n \text{ (3) } = \frac{E}{R} \times y \times dA$$

Moment of this force about N.A.

$$= \text{Force on layer} \times y$$

$$= \frac{E}{R} \times y \cdot dA \cdot y$$

$$= \frac{E}{R} \cdot y^2 \cdot dA$$

Total moment of forces (or) moment of resistance

$$= \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R} \int y^2 \cdot dA$$

Let M = external moment applied on the beam section.

For equilibrium the moment of resistance offered by the section should be equal to external bending moment.

$$\therefore \quad M = \frac{E}{R} \int y^2 . dA$$

But the expression $\int y^2 . dA$ represents the moment of inertia of the area of the section about the neutral axis.

Let Moment of Inertia be I

So, $M = \frac{E}{R} \cdot I$

$$\therefore \quad \frac{M}{I} = \frac{E}{R} \quad \text{———} \quad (3)$$

and from eqn (2)

$$\frac{\sigma}{y} = \frac{E}{R}$$

In result, $\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$ ——— (4)

The equation (4) is called bending equation.

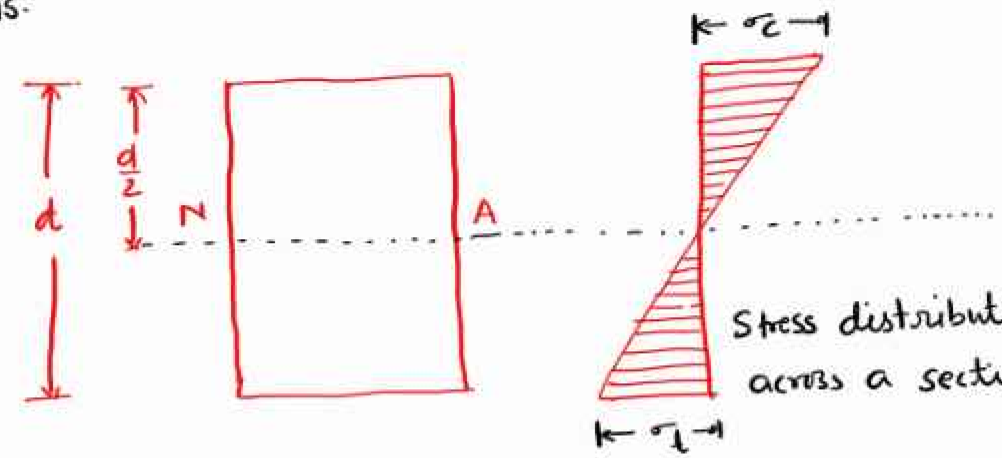
Here, M is expressed in N-mm ; I in mm^4

σ is expressed in N/mm^2 ; y in mm.

E is expressed in N/mm^2 ; R in mm.

Bending stresses in symmetrical sections

The neutral axis of symmetrical section (circular, rectangular & square) lies at a distance of $\frac{d}{2}$ from outermost layer of the section where d is the diameter (circular section) (or) depth (for rectangular (or) square) section. There is no stresses at neutral axis. But the stresses at the point is directly proportional to its distance from the neutral axis.



Stress distribution across a section

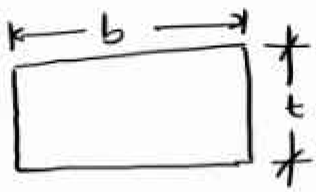
Problem: A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2\times10^5\ N/mm^2$.

Sol^n: Given data are:-

Width of plate = 120mm

Thickness of the plate = 20mm

Radius of circular arc = 10m



Moment of Inertia $= I = \frac{bt^3}{12} = \frac{120\times20^3}{12} = 8\times10^4\ mm^4$

Radius of curvature = 10m = 1000 mm

$E = 2\times10^5\ N/mm^2$

σ_{max} = Maximum stress induced

M = Bending moment

From bending equation (2)

$$\sigma = \frac{E}{R} \cdot y$$

$$y_{max} = \frac{t}{2} = \frac{20}{2} = 10\,mm$$

$$\therefore \quad \sigma_{max} = \frac{E}{R} \cdot y_{max}$$

$$\sigma_{max} = \frac{2 \times 10^5}{10000} \times 10 = 200\ N/mm^2$$

From eq^n (3)

$$\frac{M}{I} = \frac{E}{R}$$

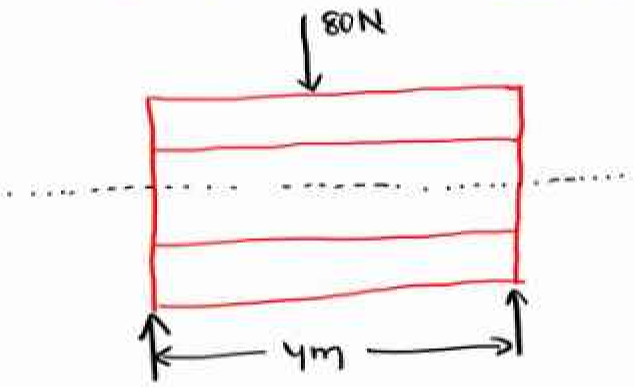
$$M = \frac{E}{R} \cdot I = \frac{2 \times 10^5}{10000} \times 8 \times 10^4$$

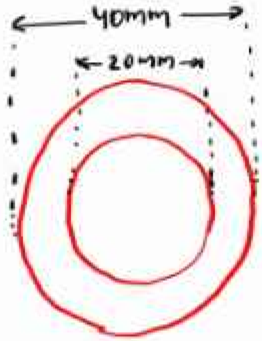
$$= 16 \times 10^5\ N\text{-}mm$$

$$\boxed{M = 1.6\ kNm}$$

Problem:

Calculate the maximum stress induced in a cast iron pipe of external diameter 40mm, of internal diameter 20mm and of length 4m. When the pipe is supported at its ends and carries a point load of 80N at its centre.





Area of cross-section

Sol^n.

Given data are:

$D = 40mm$

$d = 20mm$

$L = 4m = 4000\,mm$

$W = 80N$

So, Maximum bending moment

$$M = \frac{WL}{4} = \frac{80 \times 4000}{4} = 80000\ N\text{-}mm$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 20^4)$$

$$= 117809.7\ mm^4$$

By using equation (3)

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_{max} = \frac{M}{I} \cdot y_{max}$$

$$y_{max} = \frac{D}{2} = \frac{40}{2} = 20 mm$$

$$\sigma_{max} = \frac{8 \times 10^4}{117809.7} \times 20 = 13.58 \ N/mm^2$$

SECTION MODULUS

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

Symbol $= Z$

$$Z = \frac{I}{y_{max}}$$

$I =$ moment of Inertia about neutral axis

$y_{max} =$ Distance of the outermost layer from N.A.

From eqn (3)

$$\frac{M}{I} = \frac{\sigma}{y}$$

If σ is maximum, mean y will be maximum

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

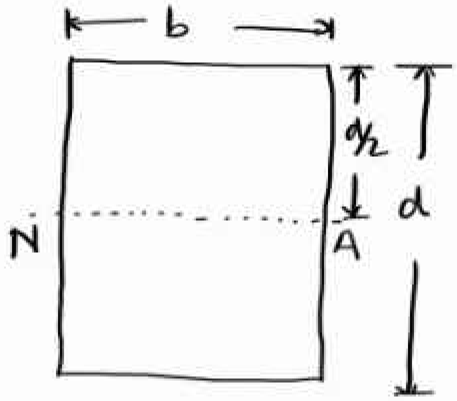
$$M = \frac{I}{y_{max}} \cdot \sigma_{max}$$

$$\boxed{M = Z \cdot \sigma_{max}}$$

So, Here, M is maximum bending moment.

SECTION MODULUS OF VARIOUS SECTION

(1) Rectangular Section



M.I. of rectangular section about Neutral axis.

$$I = \frac{bd^3}{12}$$

Distance of outermost layer from neutral axis

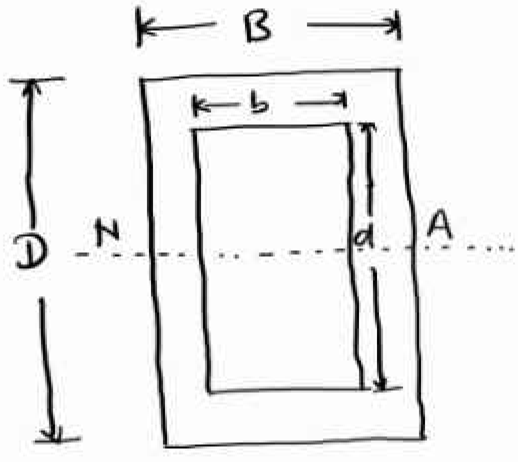
$$y_{max} = \frac{d}{2}$$

Section modulus for rectangular section

$$Z = \frac{I}{y_{max}} = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

(2) Hollow Rectangular section



$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}\left[BD^3 - bd^3\right]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

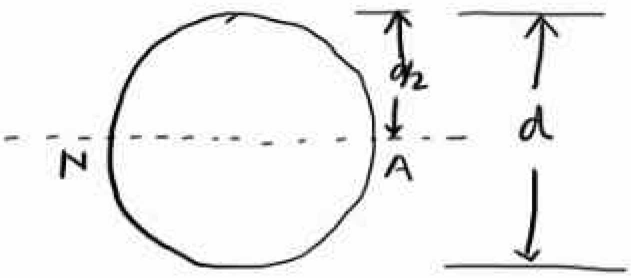
$$= \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{\frac{D}{2}}$$

$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$

(3) Circular section

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = \frac{d}{2}$$



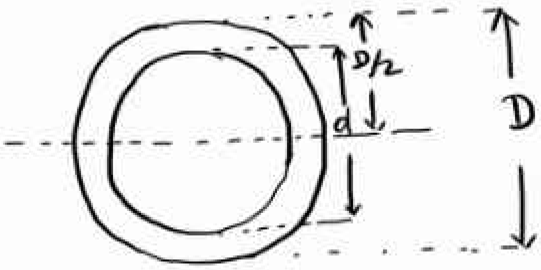
$$Z = \frac{I}{y_{max}} = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

$$\boxed{Z = \frac{\pi d^3}{32}}$$

(4) Hollow circular Section

$$I = \frac{\pi}{64} [D^4 - d^4]$$

$$y_{max} = \frac{D}{2}$$



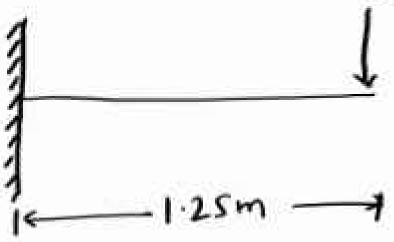
$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}[D^4 - d^4]}{D/2}$$

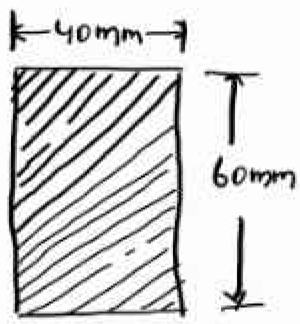
$$\boxed{Z = \frac{\pi}{32D} [D^4 - d^4]}$$

Problem:

A cantilever beam of length 1.25m fails when 3.5kN load is applied at its free end. If the section of beam is 40mm x 60mm. Find the stress at its failure.

Soln.





Here given data are.

$L = 1.25m = 1250mm$

$W = 3.5 kN = 3500 N$

$b = 40mm$

$d = 60mm$

$M = W \times L = 3500 \times 1250 = 4375000$ N-mm

Section modulus, Z

$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

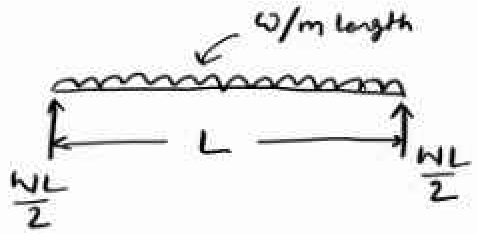
$$M = \sigma_{max} Z$$

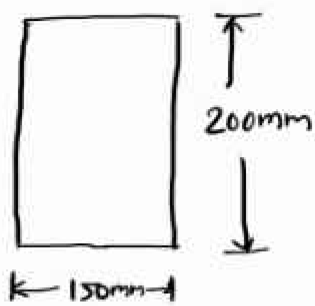
$$\sigma_{max} = \frac{M}{Z} = \frac{4375000}{24000} \text{ N/mm}^2$$

$$\sigma_{max} = \frac{4375}{24} = 182.29 \text{ N/mm}^2$$

Practice question

A rectangular beam having dimensions of 200mm deep and 150mm wide is simply supported over a span of 6m. What is uniformly distributed load per metre the beam may carry, if bending stresses is not exceeded to 120 N/mm².





Solⁿ. $d = 200mm$

$b = 150mm$

$\sigma_{max} = 120 \text{ N/mm}^2$

$L = 6m$

$$Z = \frac{bd^2}{6} = \frac{150 \times 200^2}{6} = 25 \times 40000 = 1000000$$
$$= 1 \times 10^6 \ mm^3$$

For a beam carrying udl with simply supported

$$M = \frac{\omega L^2}{8}$$

we know,

$$M = \sigma_{max} \cdot Z$$
$$M = 120 \times 1 \times 10^6 = 12 \times 10^7 \ N\text{-}mm$$
$$12 \times 10^7 = \frac{\omega \times 6000^2}{8}$$
$$\omega = \frac{12 \times 10^7 \times 8}{6000^2} = \frac{12 \times 10^7 \times 8}{36 \times 10^6}$$
$$= \frac{120 \times 8}{36} = \frac{10 \times 8}{3}$$

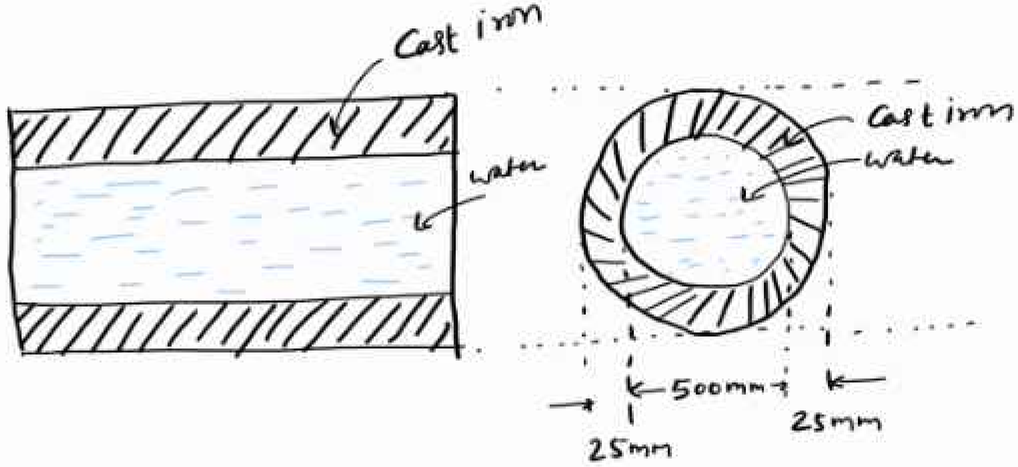
$$\omega = \frac{80}{3} = N/mm$$
$$\omega = \frac{80}{3} \times \frac{1000}{1000} = 26.67 \ kN/m$$

Problem: A square beam 20mm x 20mm in section and 2m long is supported at ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per metre length will be break a cantilever of the same material 40mm wide, 60mm deep and 3m long.?

Sol^n:

Question A Cast iron water main 12 mt long, of 500mm inside diameter and 25mm wall thickness runs full of water and is supported at its ends. Calculate the maximum stress in the metal if density of cast iron is 7200 kg/m³ and that of water is 1000 kg/m³.

Soln.



Weight of cast iron main per mt length = volume $\times \rho \times g$

$$= \frac{\pi}{4} \times (0.55^2 - 0.5^2) \times 1 \times 7200 \times 9.81$$

$$= \frac{\pi}{4} \times 0.05 \times 1.05 \times 7200 \times 9.81$$

$$= 2912.398 \text{ N}$$

Weight of water per metre length $= \frac{\pi}{4} \times 0.5^2 \times 1 \times 1000 \times 9.81$

$$= 1926.19 \text{ N}$$

Total weight of pipe per mt length $= 2912.398 + 1926.19$

$$= 4838.35 \text{ N}$$

length of pipe = 12 mt

Bending moment $= M = \frac{w l^2}{8}$

$$M = \frac{4838.35^2 \times 12^2}{8} = 87090.3 \text{ N-m}$$

$$\text{Moment of Inertia} = I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}\left[0.55^4 - 0.5^2\right]$$

$$= 1.4238 \times 10^{-3} \text{ m}^4$$

$$y_{max} = \frac{D}{2} = \frac{0.55}{2} = 0.275 \text{ m}$$

Using Bending Eqration

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_{max} = y_{max} \cdot \frac{M}{I}$$

$$= 0.275 \times \frac{87090.3}{1.4238 \times 10^{-3}}$$

$$= 16821065.11$$

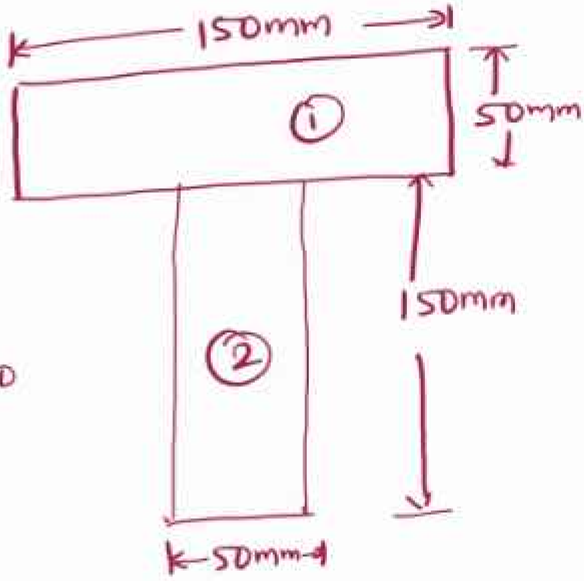
$$\boxed{\sigma_{max} = 16.82 \text{ MN/m}^2}$$

Two wooden planks 150mm x 50mm each are connected to form a T-section of a beam. If a moment of 3.4 KNm is applied around the horizontal neutral axis, inducing tension below the neutral axis, find the stresses at the extreme fibers of the cross-section. Also calculate the total tensile force on the cross-section.

Sol^n. :

Moment $= 3.4$ kN-m $= 3.4\times10^3$ N-m

	Area (a)	Centroidal distance from bottom face	ay (mm^3)
R1	150×50 =7500	175	1312500
R2	150×50=7500	75	562500
	$\Sigma a = 15000$		$\Sigma ay = 1875000$



Distance of neutral axis from bottom face

$$\bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{1875000}{15000} = 125\,\text{mm}$$

$$I_{xx} = \left[\frac{bd^3}{12} + Ay^2\right]_1 + \left[\frac{bd^3}{12} + Ay^2\right]_2$$

$$= \left[\frac{150\times50^3}{12} + 150\times50\times(175-125)^2\right] + \left[\frac{50\times150^3}{12} + 150\times50\times(125-75)^2\right]$$

$$= (1562500 + 1875000) + (14062500 + 1875000)$$

$$\boxed{I_{xx} = 5312.5\times10^4\,\text{mm}^4 \text{ (or) } 5312.5\times10^{-8}\,\text{m}^4}$$

Distance of C.G. from upper extreme fibre

$$y_c = 200-125 = 75\,\text{mm}, \quad y_t = 125\,\text{mm}$$

Using $\dfrac{M}{I} = \dfrac{\sigma}{y}$

Tensile stress, $\sigma_t = \dfrac{M}{I}\cdot y_t = \dfrac{3.4\times10^3}{5312.5\times10^{-8}}\times(125\times10^{-3})$

$= 8\times10^6 N/m^2$ or $8 MN/m^2$

Compression stress, $\sigma_c = \frac{M \times y_c}{I} = \frac{3.4\times10^3 \times 75\times10^{-3}}{5312.5\times10^{-8}}$

$= 4.8\times10^6 N/m^2$

$$\boxed{\sigma_c = 4.8 MN/m^2}$$

A timber beam of rectangular section of length 8m is simply supported. The beam carries a UDL of 15 kN/m run over the entire length and a point load of 10 kN at at 5m from the left support. If the depth is 2 times the width and the stress in the timber is not to exceed 8 N/mm², find the suitable dimension of the section.

Sol^n. length $= L = 8$ mt

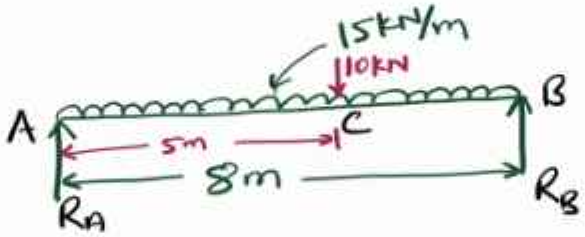
$w = 15$ kN/m

$\sigma_{max} = 8 N/mm^2$

Depth of beam $= 2 \times$ width

$= 2b$

$d = 2b$



First find the reactions

$R_A + R_B = 10 + 15\times8 = 10 + 120 = 130$

$R_A + R_B = 130$ KN

take moment about A

$R_A\times0 - 15\times8\times4 + R_B\times8 - 10\times5 = 0$

$8R_B = 10\times5 + 15\times8\times4$

$8R_B = 50 + 480 = 530$

$R_B = \frac{530}{8}$ KN $= 66.25$ KN

$R_A + R_B = 130$ KN

$R_A = 130 - \frac{530}{8} = 63.75$ KN

Here, the shear force in the beam is changing its sign between the span AC.

So, shear force between span AC will be zero.

$\therefore R_B - 15\times3 - 15(x-3) - 10 = 0$

$\frac{530}{8} - 15\times3 - 15(x-3) - 10 = 0$

$\frac{530}{8} - \cancel{45} - 15x + \cancel{45} - 10 = 0$

$66.25 - 10 - 15x = 0 \Rightarrow 15x = 56.25 \Rightarrow x = 3.75$ m

$\therefore BM_{max}$ will occur at 3.75 m from point B.

So, $M_{max} = R_B \times 3.75 - 15\times3.75\times\frac{3.75}{2} - 10\times(3.75-3)$

$= 66.25\times3.75 - 15\times3.75\times\frac{3.75}{2} - 10\times0.75$

$= 105.46875$ KNm $= 135468.750\times10^3$ Nmm

Now, Section modulus for rectangular beam

$$Z = \frac{bd^2}{6} = \frac{b\times(2b)^2}{6} = \frac{2\times2b^3}{6} = \frac{2b^3}{3}$$

Here, $M = \sigma_{max}\times Z$

$135468.750\times10^3 = 8\times\frac{2b^3}{3}$

$b^3 = \frac{135468.750\times10^3\times3}{16} \Rightarrow b = 294.95$ mm

So, $d = 2 \times b$

$= 2 \times 293.95 = 587.9\,mm$

$b = 293.95\ mm$

$d = 587.9\ mm$

Approx. & $\begin{bmatrix} b = 295\ mm \\ d = 590\ mm \end{bmatrix}$

A rolled steel joist of I-section has the dimensions as shown in fig. The I-section beam carries a udl of 40KN/m run on a span of 10m, calculate the maximum stress produced due to bending.

Sol^n.

$\omega = 40\ KN/m = 4000\ N/m$

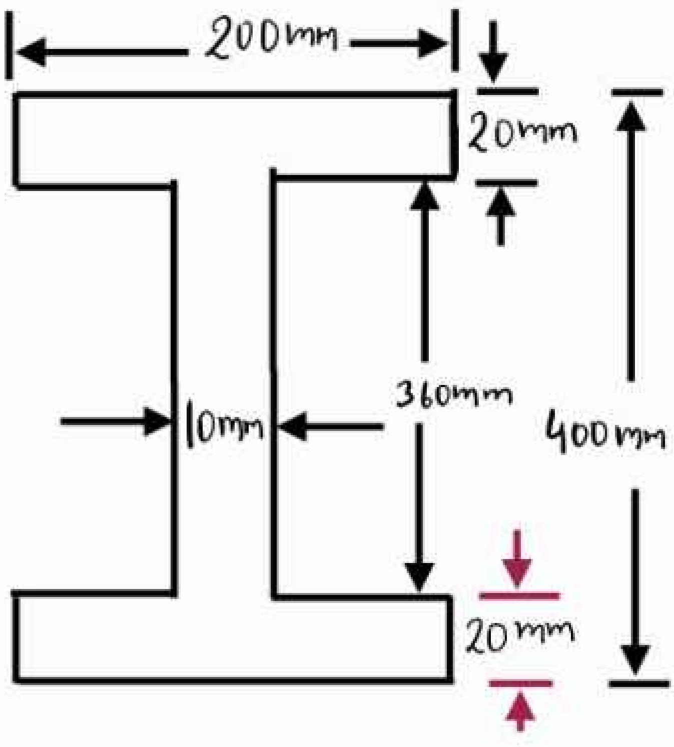
$L = 10m$

$MOI = \frac{BD^3}{12} - 2 \times \frac{bd^3}{12}$

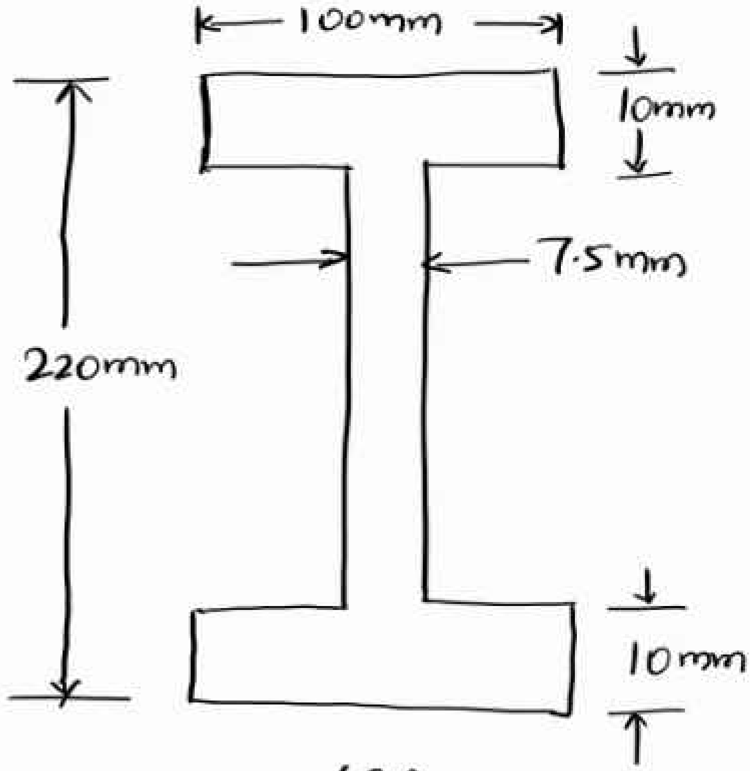
$M = \frac{\omega l^2}{8}$

Use, $\frac{M}{I} = \frac{\sigma}{y}$

get σ_{max}.



An I-section shown in figure is simply supported over a span of 12m. If the maximum permissible bending stress is $80 N/mm^2$, what concentrated load can be carried at a distance of 4m from one support.



(a)

Sol^n.

Given data is

$$\sigma_{max} = 80 N/mm^2$$

First calculate

R_A & R_B

$$R_A + R_B = W$$

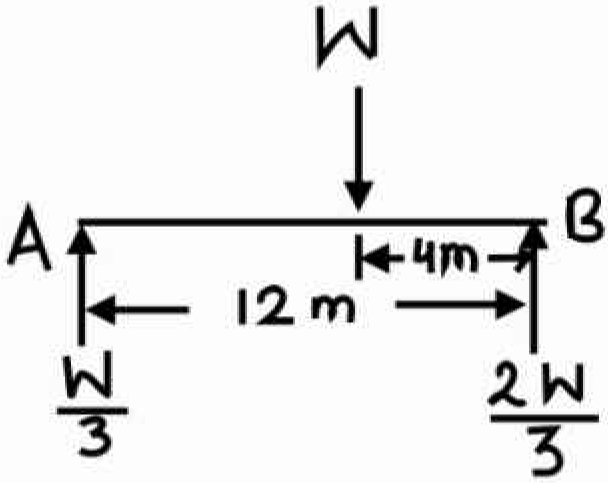
Take moment about A

$$\frac{W}{3} \times 0 - W \times 8 + R_B \times 12 = 0$$

$$12 R_B = 8W$$

$$R_B = \frac{2}{3} W$$

$$R_A = \frac{W}{3}$$



Take section CA

$$BM_x = \frac{2W}{3} x - W(x-4)$$

at point C, $x = 4m$

$$BM_C = \frac{2W}{3} \times 4 - 0 = \frac{8W}{3} \text{ N-m}$$

$$M_{max} = \frac{8W}{3} \text{ Nm}$$

Now, moment of Inertia

$$I = \frac{BD^3}{12} - 2\frac{bd^3}{12}$$

$$= \frac{100 \times 220^3}{12} - 2 \times \frac{46.25 \times 200^3}{12}$$

$$I = 27066666.67 \text{ mm}^4$$

Now, using the Bending Equations

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$y_{max} = 110 \text{ mm}$$

$$\frac{M}{27066666.67} = \frac{80}{110}$$

$$\frac{8000W}{3 \times 27066666.67} = \frac{80}{110}$$

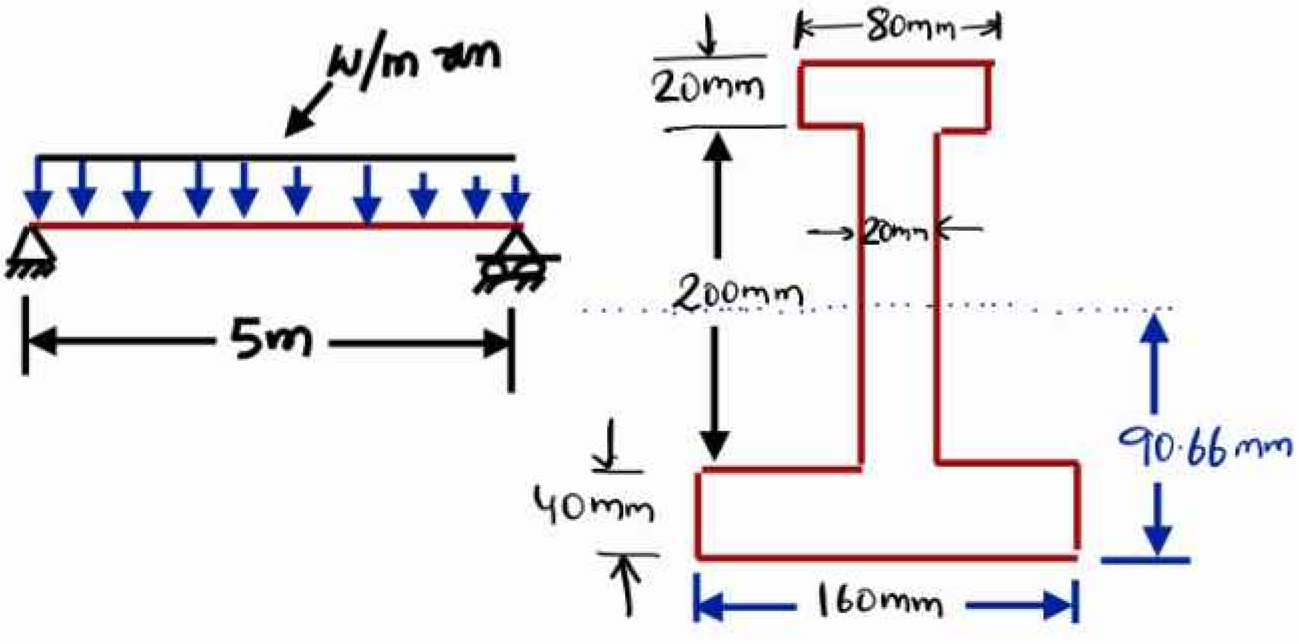
$$W = \frac{80 \times 3 \times 27066666.67}{110 \times 8000}$$

$$W = 7381.82 \text{ N} = 7.3818 \text{ kN}$$

BENDING STRESSES IN UNSYMMETRICAL SECTIONS

In the analysis of symmetrical sections, the neutral axis passes through the Geometrical centre of the section. But in case of unsymmetrical section the geometrical section will not be same and it will have different value from the top and bottom fibres.

Example: A cast iron beam is of I-section as shown in figure. The beam is simply supported on a span of 5m. If the tensile stress is not to exceed 20 N/mm². Find the safe uniformly distributed load carrying on the beam. Also, find maximum compressive stress.



Sol^n. Given data are,

$$L = 5m$$

$$\sigma_t = 20 \, N/mm^2$$

Now, find the centre of Gravity of the section.

Let $\bar{y}$ is the c.G. of section from bottom fibre.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{(A_1 + A_2 + A_3)}$$

$$= \frac{160 \times 40 \times \frac{40}{2} + 200 \times 20 \times \left(40 + \frac{200}{2}\right) + 80 \times 20 \times \left(40 + 200 + \frac{20}{2}\right)}{(160 \times 40 + 200 \times 20 + 80 \times 20)}$$

$$\boxed{\bar{y} = \frac{1088000}{12000} = 90.66 \, mm}$$

Neutral axis liest at 90.66 mm from bottom fiber and (260 − 90.66 = 169.34) from top fiber.

Now, moment of Inertia of the section about neutral axis.

Moment of Inertia about Neutral axis

$$I = I_1 + I_2 + I_3$$

I_1 = Moment of Inertia of bottom flange about N.A.

= Moment of Inertia of bottom flange about its C.G. + $A_1 \times$ (Distance of CG from NA.$)^2$

$$= \frac{160 \times 40^3}{12} + 160 \times 40 \times (90.66 - 20)^2 = 32807481.17 \text{ mm}^4$$

$$I_2 = \frac{20 \times 200^3}{12} + 200 \times 20 \times (140 - 90.66)^2 = 23071075.73 \text{ mm}^4$$

$$I_3 = \frac{80 \times 20^3}{12} + 80 \times 20 \times (250 - 90.66)^2 = 40676110.29 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3$$

$$= 32807481.17 + 23071075.73 + 40676110.29$$

$$\boxed{I = 96554667.21 \text{ mm}^4}$$

In the given section,

The maximum tensile stress will be at bottom extreme fiber and compressive stress at top extreme fibre.

Here, maximum tensile stress is given as 20 N/mm^2.

So, y_{max} will be considered from bottom fiber because max. tensile stress will be at bottom fiber.

So, $y_{max} = 90.66$ mm

Using Bending eqn

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma}{y} \cdot I$$

$$= \frac{20}{90.66} \times 96554667.21 = 21300389.85 \text{ N-mm} \quad \text{—} ①$$

As beam is simply supported with UDL

So, $$BM_{max} = \frac{wL^2}{8}$$

$$= \frac{w \times 5^2}{8} = \frac{w \times 25 \times 1000}{8} = 3125w \text{ N-mm} \quad \text{—} ②$$

Equating two bending moment values

$$3125w = 21300389.85$$

$$\boxed{w = \frac{21300389.85}{3125} = 6816.125 \text{ N/m}}$$

Now, maximum compressive stress

Distance of N.A. from extreme top fiber

$y_c = 169.34$ mm

$M = 21300389.85$

$I = 96554667.21$

Using Bending Equation

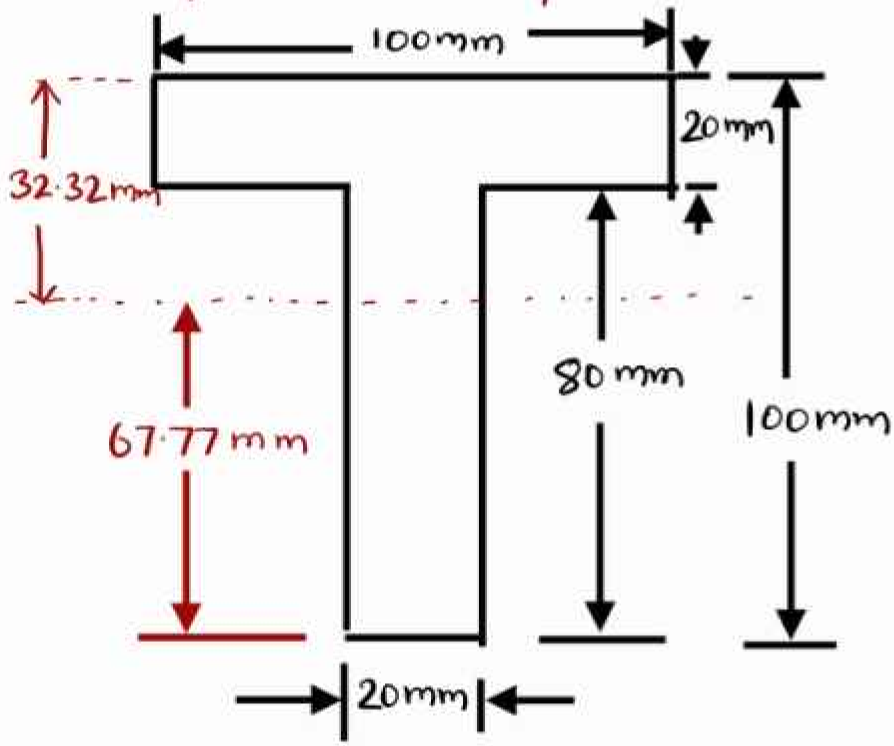
$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma_c = \frac{M}{I} \cdot y_c$$

$$= \frac{21300389.85}{96554667.21} \times 169.34$$

$$\boxed{\sigma_c = 37.357 \text{ N/mm}^2}$$

A Cast iron beam of T-section as shown in figure. The beam is simply supported on a span of 8m. The beam carries a UDL of 1.5 kN/m length on the entire span. Determine the maximum tensile and compressive stresses.

Soln.



Given data are

Length = 8 m

ω = 1.5 kN/m = 1500 N/m

Position of N.A is calculated with the help of calculation of C.G. Let $\bar{y}$ distance of C.G. from bottom.

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{100 \times 20 \times \left(80 + \frac{20}{2}\right) + 80 \times 20 \times 80/2}{(100 \times 20) + (80 \times 20)}$$

$$= \frac{244000}{3600} = 67.77 \text{ mm}$$

Neutal axis lies at a distance of 67.77mm from the bottom face. and $(100 - 67.77 = 32.32 \text{ mm})$ from top face.

Moment of Inertia of the section about N.A.

$$I = I_1 + I_2$$

I_1 = Moment of Inertia of top flange about N.A.

= MOI. of top flange about C.G. + $A_1 \times$ (Distance of C.G from N.A)^2

$$= \frac{100 \times 20^3}{12} + 100 \times 20 \times \left(32.23 - \frac{20}{2}\right)^2$$

$$= \frac{100 \times 20^3}{12} + 100 \times 20 \times 22.23^2 = 1055012.467 \text{ mm}^4$$

$$I_2 = \frac{80^3 \times 20}{12} + 80 \times 20 \times (67.77 - 40)^2$$

$$= \frac{20 \times 80^3}{12} + 80 \times 20 \times 771.1729 = 2087209.973 \text{ mm}^4$$

$$I = I_1 + I_2 = 1055012.467 + 2087209.973$$

$$= 3142222.4 \text{ mm}^4$$

For Simply supported beam tensile stress will be at extreme bottom fibre and compressive stress at extreme top fibre.

$$BM = \frac{wl^2}{8} = \frac{1500 \times 8^2}{8} \text{ N-m}$$

$$= 12000000 \text{ N-mm} = 12 \times 10^6 \text{ N-mm}$$

Using Bending Equation

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M}{I} \cdot y$$

(1) For maximum tensile stress

$$y_{max} = 67.77\,mm$$

$$\sigma_t = \frac{M}{I} \cdot y_{max} = \frac{12\times10^6}{3142222.4} \times 67.77 = 258.81\ N/mm^2$$

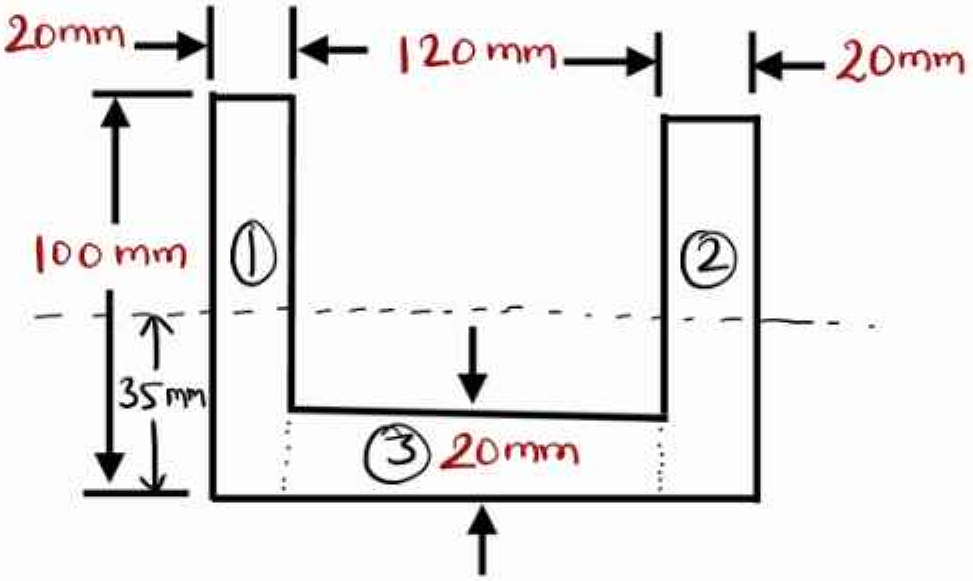
(2) For maximum compressive stress

$$y_{max} = 32.23\,mm$$

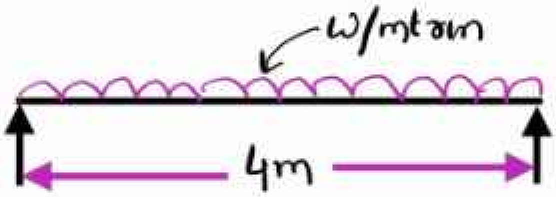
$$\sigma_c = \frac{M}{I} \cdot y_{max} = \frac{12\times10^6}{3142222.4} \times 32.23$$

$$= 123.08\ N/mm^2$$

\# The horizontal beam of section Shown in figure. is 4m long and is simply supported at the ends. Calculate the maximum Uniformly distributed load it can Carry if the tensile and compressive Stresses must not exceed 20 MN/m^2 and 40MN/m^2 respectively.



Sol^n.



$\sigma_t = 20 MN/m^2$

$\sigma_c = 40 MN/m^2$

Calculate the centroid of channel section

take reference as bottom fiber.

$A_1 = 100 \times 20 = 2000 \ mm^2$

$A_2 = 100 \times 20 = 2000 \ mm^2$

$A_3 = 120 \times 20 = 2400 \ mm^2$

$$\text{Centroid of Channel} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{2000 \times 50 + 2000 \times 50 + 2400 \times 10}{2000 + 2000 + 2400}$$

$$= 35 \ mm \text{ from bottom}$$

So, Distance of neutral axis from bottom $= 35 mm$

and Distance of neutral axis from top $= 100 - 35 = 65 mm$

Moment of Inertia = $I = I_1 + I_2 + I_3$

$$I_1 = \left(\frac{bd^3}{12}\right)_1 + A_1 \bar{y}_1^2$$

$$= \frac{20 \times 100^3}{12} + 2000 \times (50-35)^2$$

$$\boxed{I_1 = 2116666.667 \text{ mm}^4}$$

$$I_2 = \left(\frac{bd^3}{12}\right)_2 + A_2 \bar{y}_2^2$$

$$= \frac{20 \times 100^3}{12} + 2000 \times (50-35)^2$$

$$\boxed{I_2 = 2116666.667 \text{ mm}^4}$$

$$I_3 = \left(\frac{bd^3}{12}\right)_3 + A_3 \bar{y}_3^2$$

$$= \left(\frac{120 \times 20^3}{12}\right) + 2400 \times (35-10)^2$$

$$\boxed{I_3 = 1580000 \text{ mm}^4}$$

So, $I = I_1 + I_2 + I_3 = 2116666.667 + 216666.667 + 1580000$

$$\boxed{I = 5813333.334 \text{ mm}^4}$$

Bending Moment of the beam with simply supported carrying UDL

$$M = \frac{wl^2}{8} = \frac{w \times (4 \times 1000)^2}{8}$$

$$= \frac{w \times 16 \times 10^6}{8}$$

$$\boxed{M = 2 \times 10^6 w \text{ N-mm}}$$

Using Bending theory.

$$\frac{M}{I} = \frac{\sigma_t}{y_t} = \frac{\sigma_c}{y_c}$$

$$\frac{\sigma_c}{\sigma_t} = \frac{y_c}{y_t} = \frac{65}{35}$$

$$\frac{\sigma_c}{\sigma_t} = 1.857$$

$$\sigma_c = 1.857\,\sigma_t$$

Permissible limit of $\sigma_c = 1.857 \times 20$

$$= 37.14 \text{ MN/m}^2$$

$$= 37.14 \times 10^6 \text{ N/m}^2$$

$$\sigma_t = \frac{\sigma_c}{1.857} = \frac{40}{1.857} = 21.54 \text{ MN/m}^2$$

$$= 21.54 \times 10^6 \text{ N/m}^2$$

$$= 21.54 \text{ N/mm}^2$$

So, to calculate load.

$$\frac{M}{I} = \frac{\sigma_t}{y_t}$$

$$\frac{2\times10^6 \omega}{5183333.334} = \frac{21.54}{35}$$

$$\omega = \frac{21.54 \times 5183333.334}{2\times35\times10^6}$$

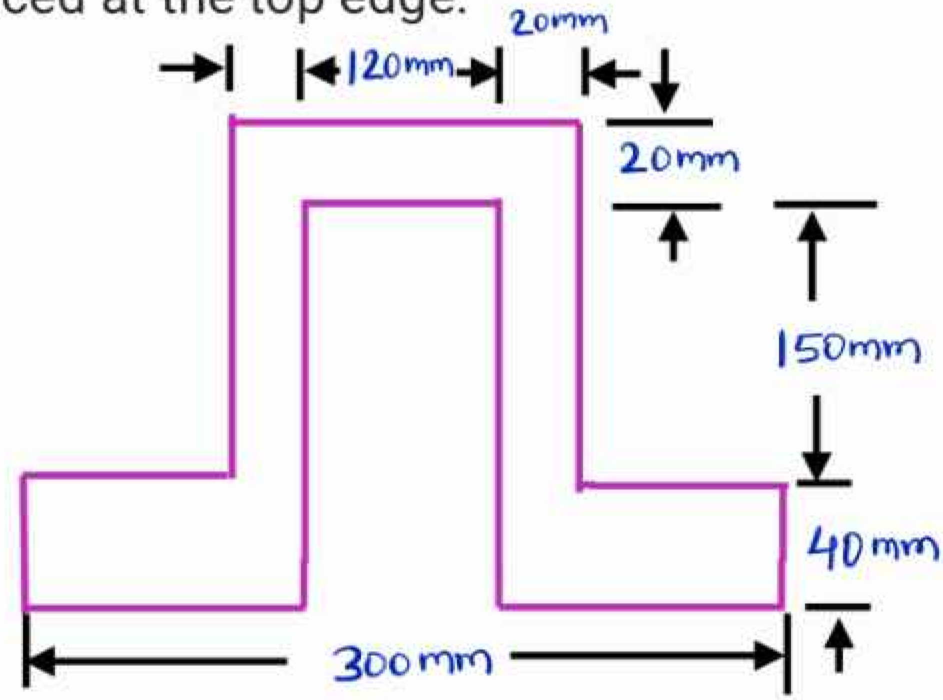
$$\omega = 1594985714 \ N/mm$$

$$\boxed{\omega = 1.595 \ kN/m}$$

A cast iron beam cross-section is given in figure. When this beam is subjected to bending moment the tensile Strees at the bottom edge is $30 MN/m^2$. Calculate

1 The valve of bending moment

2 Stress induced at the top edge.



Solⁿ.

(i) Centroid → $\bar{y}$

(ii) MOI

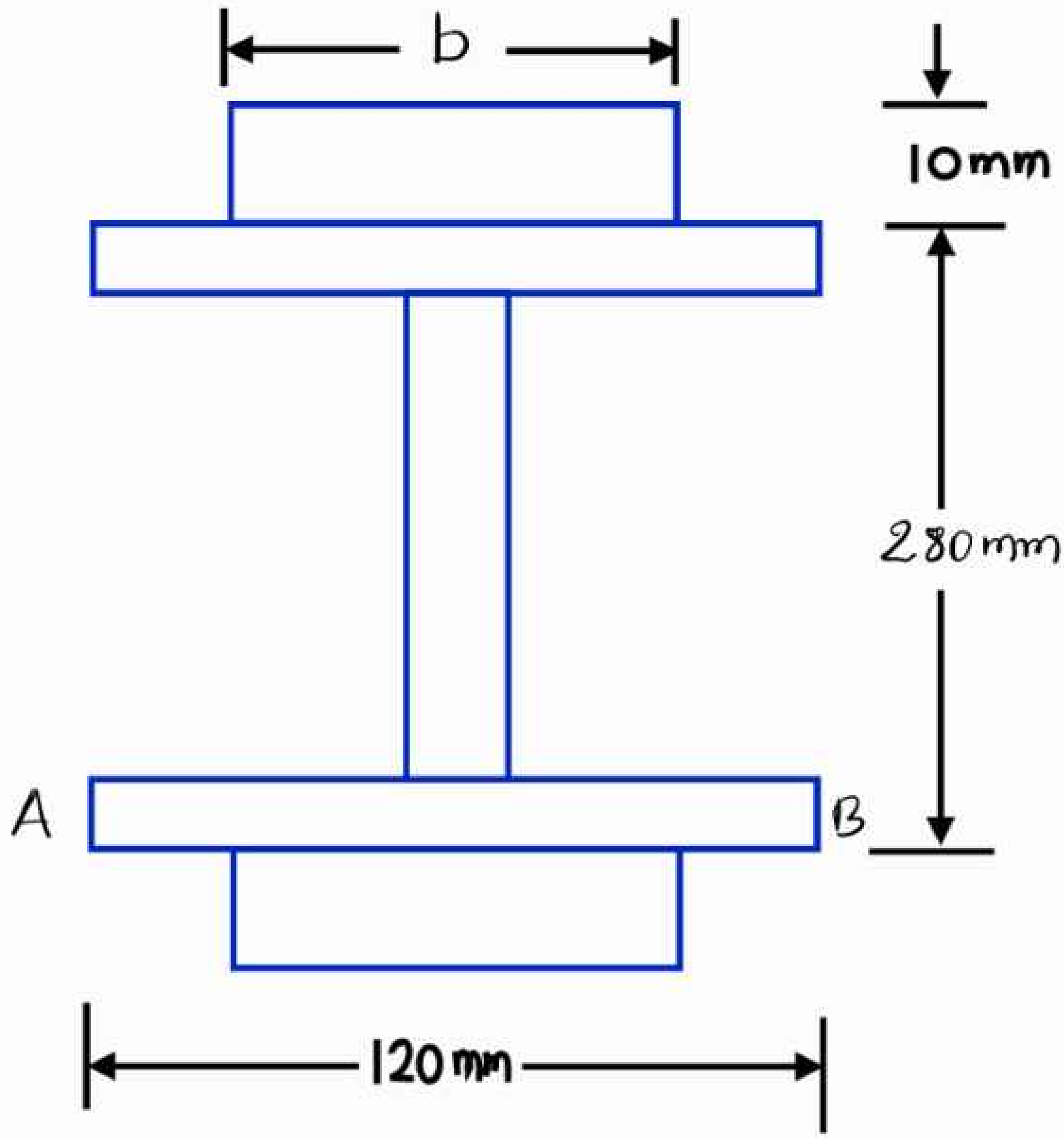
(iii) M

(iv) σ_c

A 280mm × 120mm I-section beam is to be used as a cantilever of 3.6 mt length. Find the uniformly distributed load which can be carried by the beam, if the permissible stress is 120 MPa and $I = 75 \times 10^6 \, mm^4$.

If the cantilever is strengthened by 10mm thick steel plates welded at the top and bottom flanges to withstand a 40% increase load, find the width of the plates and the length over which the plate should extend, the maximum stress being the same.

Sol^n.



Given $I = 75 \times 10^6 \ mm^4$

$\sigma = 125 \ MPa$

Load is increase by 40% due to addition of plate with 10mm thickness.

Moment of Resistance, $M = \frac{\omega l^2}{2}$

$$= \frac{\omega \times 3.6^2}{2}$$

$$= 6.48\omega \text{ N-m}$$

$$= 6480\omega \text{ N-mm}$$

Here, $y_{max} = 140mm$

Using Bending theory

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow \frac{6480\omega}{75 \times 10^6} = \frac{125}{140}$$

$$\Rightarrow \omega = 10334 \text{ N/m}$$

$$\therefore M = 6480 \times 10334 = 66964 \text{ N-m}$$

When load increased by 40%, then

$$\omega = 10334 \times 1.40 = 14468 \text{ N/m}$$

and y_{max} will increase

as $y_{max} = 140 + 10 = 150mm$

Now, increased bending moment

$$M = \frac{wl^2}{2} = \frac{14468 \times 3.6^2}{2} = 93.75 \times 10^6 \text{ N-mm}$$

Now,

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\frac{125}{150} = \frac{93.75 \times 10^6}{I}$$

$$I = 112.5 \times 10^6 \text{ mm}^4$$

Now, increase in MOI $= 112.5 \times 10^6 - 75 \times 10^6$

$$= 37.5 \times 10^6 \text{ mm}^4$$

Calculation of width and length of the plate

$$2 \times \left(\frac{b \times 10^3}{12} + b \times 10 \times 145^2 \right) = 37.5 \times 10^6$$

$$\frac{b \times 10^3}{6} + 20b \times 145^2 = 37.5 \times 10^6$$

$$b\left(\frac{10^3}{6} + 20 \times 145^2 \right) = 37.5 \times 10^6$$

$$b = \frac{37.5 \times 10^6}{420666.6667} = 89.14 \text{mm}$$

For the increased load BM reaches maximum permissible value 66964 N-m at a distance of 'x' from free end.

$$M = \frac{wx^2}{2} = \frac{14468 \times x^2}{2}$$

$$66964 = \frac{14468\, x^2}{2}$$

$$x^2 = \frac{2 \times 66964}{14468}$$

$$x = \sqrt{\frac{2 \times 66964}{14468}} = 3.04\, m$$